DEPARTMENT OF CIVIL, ENVIRONMENTAL AND MECHANICAL ENGINEERING

Discretization of the Grain Size Distribution and its effects on river morphodynamics modeling

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Introduction

River beds composition is usually characterized by the presence of mixed sediment. To describe the interactions between different grains, numerical modeling of river morphodynamics relies on the mathematical description of mixed-sediment morphodynamics. This is usually implemented in numerical models using the model by Hirano (1971, 1972), where the grain size distribution (GSD) is discretized using a finite number of classes, each characterized by a representative grain-size and fraction. Despite the large number of applications of the Hirano model, it remains unclear how many many classes are needed to properly discretize a GSD and how the GSD should be discretized.

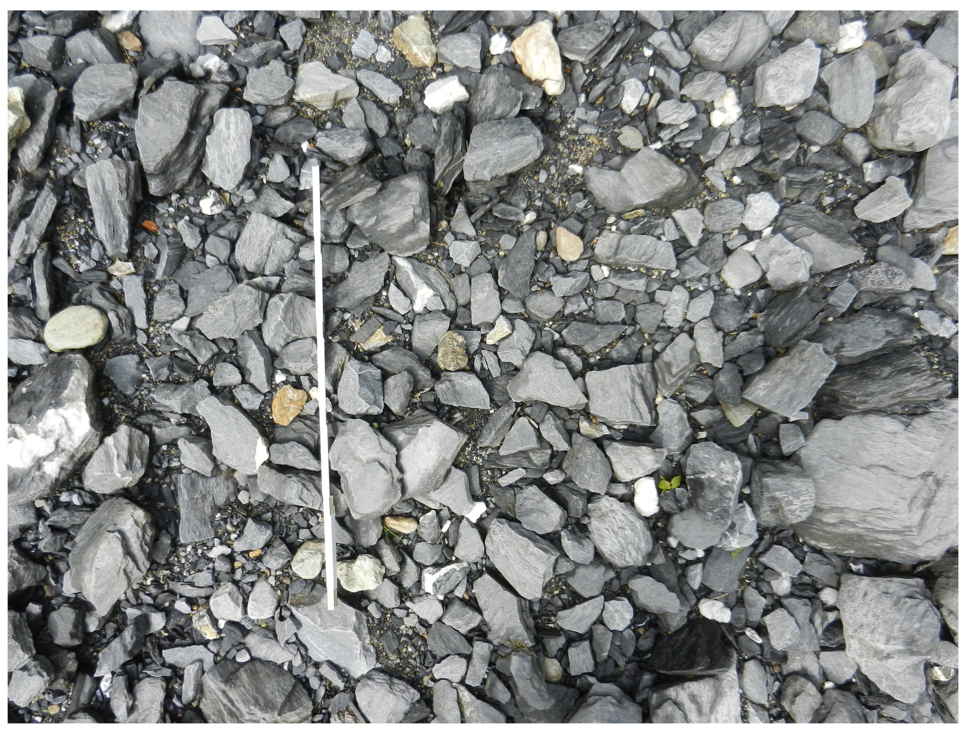

Fig. 1. Grain size distribution (GSD) in a gravel-bed river (Albula River, Canton of Grisons, Switzerland)

Objectives & methods

The **main aim** of this work is to underline the effects of different GSD discretization methods on the numerical modeling of river morphodynamics.
To quantify these effects, we run 2D numerical simulations with BASEMENT (www.basement.ethz.ch). The model describes the hydro-dynamics by the Saint-Venant equations. Friction exerted by flow over a cohesionless bottom composed of mixed sediment induces sediment transport, which is assumed to occur only as bedload. The GSD of the riverbed surface and the development of size stratification are described using the active-layer approach of Hirano (Hirano 1971, 1972).

GSD discretization

Grain size is often specified in terms of a base-2 logarithmic scale (phi-scale or psi-scale) where:

$$\psi = -\phi = log_2\,(d)$$

Each grain class is characterized by a volume fraction f_k and a grain size d_k, and it is defined by two fractions ($f_{k-1/2}$ and $f_{k+1/2}$) and two sizes ($d_{k-1/2}$ and $d_{k+1/2}$). N_{gs} grain sizes (bounds) define $N_{gc} = N_{gs} - 1$ grain classes, and f_k and d_k can be calculated following

$$f_k = f_{k+1/2} - f_{k-1/2}$$

$$d_k = (d_{k+1/2}\, d_{k-1/2})^{1/2}$$

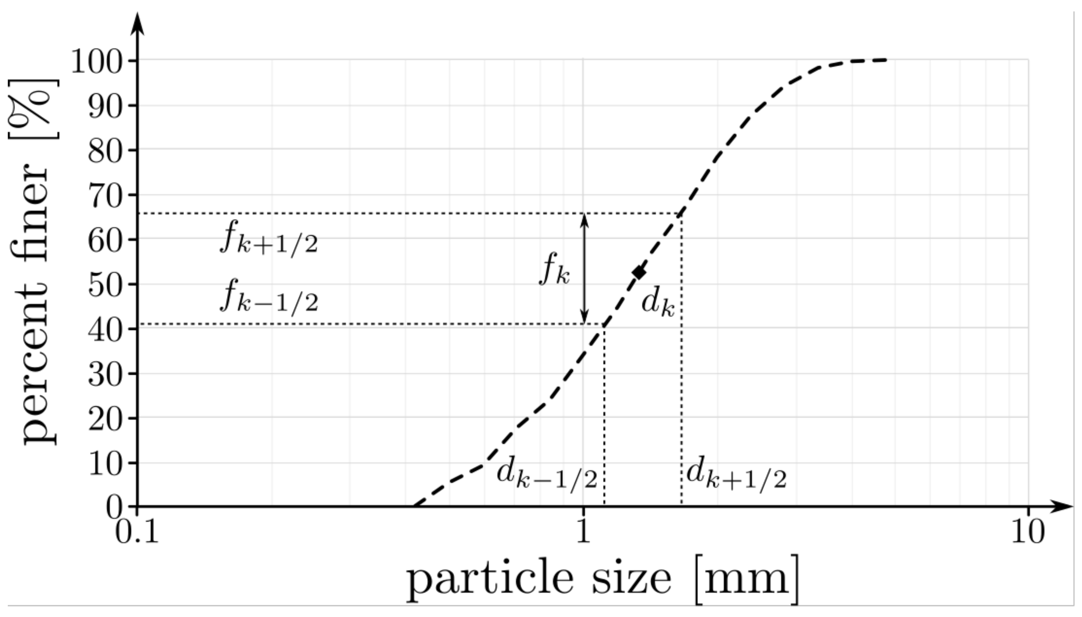

Fig. 2. Representation and discretization of a grain size distribution

GSDs can be discretized either by subdividing the diameters range, or by identifying proper fractions. In the first case, (i) the number of grain classes N_{gc} is set, (ii) the bounds are interpolated from the original GSD starting from the first diameter of the initial distribution, and (iii) d_k and f_k are calculated for each class. In the second case, (i) the desired frequencies are set, and (ii) d_k values are interpolated from the original GSD for each class.

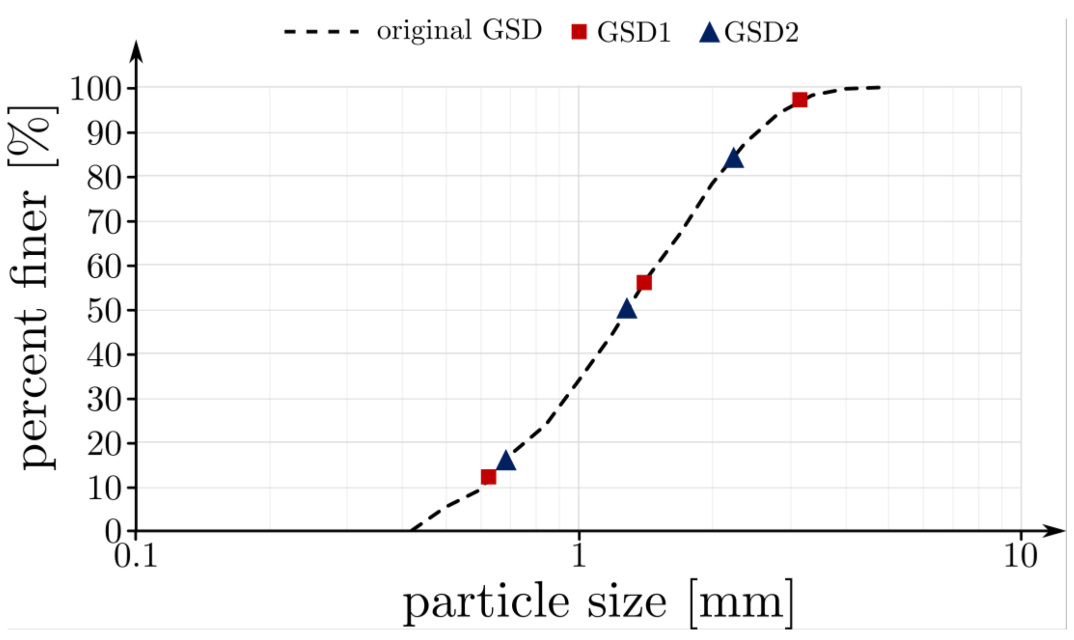

Fig. 3. Result of two different discretization methods, one based on the division of the diameter range (GSD1) and one based on the choice of percent (GSD2)

The original GSD used for the reference laboratory runs is characterized by a d_{50} = 1.29 mm and a mean geometric size d_g = 1.26 mm. In Figure 3 the results of two different methods for discretizing the GSD are represented. The original GSD is discretized with 3 grain classes (i) subdividing the diameters range into 3 equal intervals (GSD1), and (ii) choosing three frequencies to discretize the GSD with three characteristic diameters, i.e. the d_{16}, d_{50} and d_{84} (GSD2). The characteristic sizes of GSD1 in Figure 3 correspond to the d_{12}, d_{56} and d_{97}.

Numerical model setup

The model setup and calibration is supported by and validated against laboratory experiments, which provided accurate data on bed topography, surface texture, and bedload flux. The numerical domain refers to a laboratory flume with a length of 22.5 m and an initial slope of 1%. The cross-sectional geometry is rectangular with a constant width of 0.38 m. The channel is discretized using an unstructured grid composed by more than 16k triangles. Water and bedload are fed at the upstream end of the domain. They are characterized by constant values Q_w and Q_b which are given in Table 1, together with the duration of each run.

	(a)	(b)	(c)
Q_w [m³/s]	1.7E-3	3.0E-3	4.2E-3
Q_b [m³/s]	2.19E-8	1.13E-7	2.42E-7
t_{end} [s]	277800	115200	3600

Table 1. Summary of input Q_w and Q_b and duration for numerical simulations.

Results

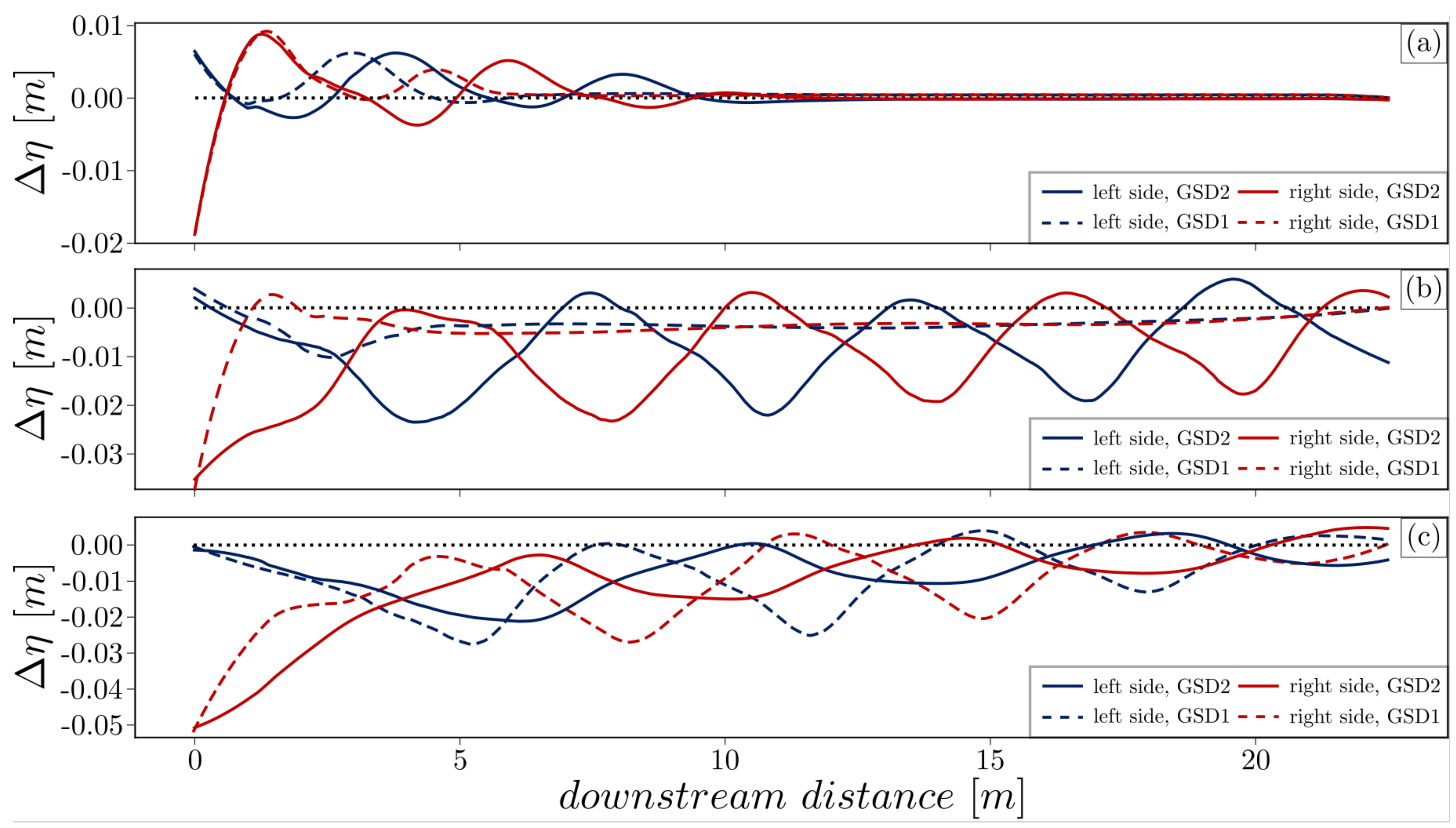

Fig. 4. Results concerning the development of alternate bars. Plots refer to the final stages of simulations with an upstream water discharge (a) Q_w = 0.0017 m³/s, (b) Q_w = 0.003 m³/s, (c) Q_w = 0.0042 m³/s (see Table 1).

Results shown in Figure 4 are relative to the final stage of numerical runs performed with constant water discharge Q_w = 0.0017 m³/s (Fig.4 (a)), Q_w = 0.003 m³/s (Fig.4 (b)), and Q_w = 0.0042 m³/s (Fig.4 (c)). The downstream distance is shown on the x-axis and results are given in terms of elevation difference Δη. Blue and red lines represent the left and right side of the channel (0.04 m from the wall), respectively. Solid lines are relative to GSD2 (blue triangles in Figure 3) and dashed lines refer to GSD1 (red squares in Figure 3).
Results show that:

1. with Q_w = 0.0017 m³/s (Fig.4 (a)) bars start to form at the final stage of the simulation both for GSD1 and GSD2;
2. bars do not form with Q_w = 0.003 m³/s (Fig.4 (b)) with GSD1;
3. bars are present over the whole length of the reach with Q_w = 0.0042 m³/s (Fig.4 (c)), but they are longer for GSD2 than for GSD1.

The riverbed composition (not shown here) present similar patterns, that is:

1. with Q_w = 0.0017 m³/s the effect of the feeding is confined within the first 5 km for GSD1 while it reaches 10 km for GSD2;
2. with Q_w = 0.003 m³/s the coarsest grains accumulate at the upstream end of the channel;
3. with Q_w = 0.0042 m³/s bars get longer for GSD2 than for GSD1 and the riverbed surface composition gets more uniform.

Conclusions

The discretization of the GSD is an important process for the modeling of river morphodynamics. The two methods presented here to discretize the GSD can be both used depending on the processes that are under investigation. Generally speaking, the results presented here show that:

- discretizing the GSD by dividing the diameters range (GSD1, Fig.3) produces coarser distributions;
- the presence of very coarse grains (i.e. d_{97}) slows down the morphodynamics and can possibly suppress the formation of bed forms;

References

Hirano, M. (1971), River bed degradation with armoring, Transactions of the Japan Society of Civil Engineers, 3(2), 194–195.
Hirano, M. (1972), Studies on variation and equilibrium state of a river bed composed of non-uniform material, Transactions of the Japan Society of Civil Engineers, 4, 128–129.

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